Global Oscillation Network Group

Report Number 14

SOLAR MODEL PROJECT
File format for
GONG solar model project.

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1. Introduction.

The present notes provide a definition of the data format used for comparison of solar models within the GONG models team. They essentially correspond to Section 9 of Computational procedures for GONG model project (in the following CP) and have been extracted here for convenience of reference. For details on the GONG model comparisons project, involving simplified physics, CP should be consulted. The focus here is on the exchange of realistic solar models, involving detailed physics.

2. File structure, and variables

The exchange of models will take place by means of ASCII files, to avoid problems with different binary formats. This makes it possible to send the data electronically without problems (other than caused by the amount).

For each model the file consists of a header, with descriptive information, a set of global variables, and a set of variables given at each mesh point.

The set of variables is likely to develop, particularly in the context of the use of the format for transferring realistic solar and stellar models. The actual set used is defined by the variable ivers included in the header to the file. In addition, the total number of global parameters, meshpoints and variables at each meshpoint are given in the header, so that the information required to read the file is available.

The variables should be given in cgs units, unless otherwise stated.

2.1 Text header.

The first record should contain the name of the model, its date, and an identification of its origin (such as name or institute). In addition the header may contain text further describing the calculation, and information about the remaining data.

An example of a header may be the following (describing Model S of the Science series of articles):

L5BI.D.15.PRES.950912.AARHUS
Level 5 physics, present Sun. (OPAL opacity, OPAL EOS). He, Z diffusion.
Age of present Sun: 4.6 Gyr.

2.2 Global parameters.

These are set up in the array glob(i), i = 1, ..., iconst, with the following definition:

1: M (total mass).
2: R (photospheric radius).
3: $L_s$ (surface luminosity).
4: $Z$ (heavy element abundance).
5: $X_0$ (initial hydrogen abundance).
6: $\alpha = \ell/H_p$ (mixing-length parameter; $H_p$ is pressure scale height)
7: $\phi$ (another convection theory parameter)$^a$
8: $\xi$ (yet another convection theory parameter)$^a$
9: $\beta$ (parameter in surface pressure condition)$^b$
10: $\lambda$ (parameter in surface luminosity condition)$^b$

11: $\frac{R^2 d^2 p_c}{p_c} \text{ (at centre)}^c$
12: $\frac{R^2 d^2 \rho_c}{\rho_c} \text{ (at centre)}^c$

13: Model age (in years)$^d$

14 - 15: Unused, so far.

Notes:

a) The parameters $\phi$ and $\xi$ are defined in CP. For the Böhm-Vitense formulation, $\phi = 9/4$ and $\xi = 1/162$.
b) These parameters are used in boundary conditions for models with simplified physics (cf. CP).
   For realistic models, $\beta = \lambda = 1$.
c) These second derivatives may be needed in the central boundary conditions for oscillation calculations.
d) Added 30/7/96. If the calculation includes pre-main-sequence evolution, the definition of zero age must be specified, in the character header or in accompanying notes. It would be useful to arrive at a common definition; suggestions are welcome.

2.3 Model variables at each mesh point.

These are set up in the array $\text{var}(i, n)$, $i = 1, \ldots, \text{ivar}$, $n = 1, \ldots, \text{nn}$. The current set of variables, as defined in July 1996, is characterized by having $\text{ivers} = 250$.

1: $r$ (distance to centre)
2: $\ln q$, $q = m/M$ ($m$ is mass interior to $r$ and $M$ is total mass)
3: $T$ (temperature)
4: $p$ (pressure)
5: $\rho$ (density)
6: $X$ (hydrogen abundance by mass)
7: $L(r)$ (luminosity at distance $r$ from centre)
8: $\kappa$ (opacity)
9: $\epsilon$ (energy generation rate per unit mass)
10: $\Gamma_1 = \left( \frac{\partial \ln p}{\partial \ln \rho} \right)_{ad}$
11: $\nabla_{ad} = \left( \frac{\partial \ln T}{\partial \ln p} \right)_{ad}$
12: $\delta = - \left( \frac{\partial \log \rho}{\partial \log T} \right)_p$
13: $c_p$ (specific heat at constant pressure)
14: $\mu_e^{-1}$ [see note ii) below]
15: \[
\frac{1}{\Gamma_1} \frac{d \log p}{d \log \rho} - \frac{d \log \rho}{d \log r}
\]
16: $r_X$ (rate of change in X from nuclear reactions)
17: $Z$ (heavy-element abundance per unit mass)
18: $R - r$
19: $\epsilon_g$ (rate of gravitational energy release)
20: $L_g$ (local gravitational luminosity; this has only been included in J. Reiter's models so far)
21: $X(^3\text{He})$ ($^3\text{He}$ abundance by mass)
22: $X(^{12}\text{C})$ ($^{12}\text{C}$ abundance by mass)
23: $X(^{13}\text{C})$ ($^{13}\text{C}$ abundance by mass)
24: $X(^{14}\text{N})$ ($^{14}\text{N}$ abundance by mass)
25: $X(^{16}\text{O})$ ($^{16}\text{O}$ abundance by mass)
26: \[
\left( \frac{\partial \ln \Gamma_1}{\partial \ln \rho} \right)_{p,Y}
\]
27: \[
\left( \frac{\partial \ln \Gamma_1}{\partial \ln p} \right)_{\rho,Y}
\]
28: \[
\left( \frac{\partial \ln \Gamma_1}{\partial Y} \right)_{p,\rho}
\]
29 – 30: Currently not used.

Comments:

i) In variable 2, ln $q$ is used instead of $m$ to give a better indication of variation close to the surface. Here “ln” is natural logarithm.

ii) $\mu_e^{-1} = N_e m_u$, where $N_e$ is the number of free electrons per unit mass and $m_u$ is the atomic mass unit; thus $\mu_e$ is the mean molecular weight per electron. It has been included to give some indication of the ionization state.

iii) Variable 18 was introduced Aug. 6 1993, to avoid problems with interpolation in $r$ near surface. (For versions before vers = 210, but after that date, $R - r$ was set in variable 17.)

iv) The derivatives of $\Gamma_1$ were introduced principally to allow calculation of kernels involving $Y$, in realistic models; they are less essential for the comparison of models.

v) Calculations that do not follow all details of the CNO cycle may not have available some of the CNO abundances. In that case, the corresponding abundances can be set to zero. (An example might be a calculation following the conversion of $^{16}\text{O}$ into $^{14}\text{N}$ but ignoring the details of carbon burning into $^{14}\text{N}$.)

vi) More generally, for a given code there might be variables that are difficult to obtain. In that case, the above numbering should be maintained, but the missing variables may be set identically to zero.

The present set of variables has been chosen to give a reasonably comprehensive basis for comparing evolution models, and to be adequate for the computation of adiabatic oscillations. In particular these variables should define completely the stellar structure equations, and so permit a check of the accuracy to which the equations are satisfied. For a more detailed comparison even more variables may be needed (such as ionization levels), but I suggest that that be arranged separately.
For non-adiabatic calculations, which we should eventually get to, more variables are certainly needed. Typical examples are the derivatives of $\kappa$ and $\epsilon$ with respect to $p$ and $T$, and possibly variables relating to the perturbation of the convective flux. They can be included later, by extending the basic set given above.

2.4 Format for data transfer.

The data exchange should be carried out by means of formatted ASCII files, using the following structure:

- **Record 1**: Name of model (as a character string)
- **Record 2 - 4**: Explanatory text, in free format
- **Record 5**: $nn$, $iconst$, $ivar$, $ivers$
- **Record 6 - 8**: glob($i$), $i = 1, \ldots, iconst$
- **Record 9**: $: var(i,n)$, $i = 1, \ldots, ivar$, $n = 1, \ldots nn$.

Here $nn$ is the number of mesh points in the model, $iconst$ is the number of global variables (given in the array glob), and $ivar$ is the number of variables at each mesh point (given in the array var). Thus with the present set $iconst = 15$ and $ivar = 30$, but there is room for expansion. The version number $ivers$ was discussed above.

The integer variable Record 5 should be written with the format

4i10

and the real variable Records 6 and up with the format

1p5e16.9

3. Summary of earlier versions

The format has been in used for almost a decade, with various extensions or modifications; these are characterized by the version number, as summarized below:

- **Version 100**: $var(1) - var(16)$ defined as above. In some models $var(17)$ was set to $R - r$, and $var(19)$ and $var(20)$ were set as above; otherwise these variables were set to zero; $ivar = 20$.
- **Version 200**: $var(1) - var(16)$ and $var(19) - var(25)$ defined as above, $var(17)$ set to $R - r$; $ivar = 25$.
- **Version 210**: $var(1) - var(25)$ defined as above; $ivar = 25$. 

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Computational procedures for GONG solar model project.

3. edition, July 1996

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Notes on revision: The original version of these notes dates from 1988. In the present version the output format has been extended, to include more information about the abundances and derivatives $\Gamma_1$. Some comments about the changes relative to the earlier version are given in the Appendix.

1. Introduction.

The present notes give details on the computational procedures to be used for the comparative computation of solar models within the GONG Solar Model Team. This exercise was described in some detail in GONG Newsletter No. 6, with further information on the progress so far given in GONG Newsletter No. 7. However, for completeness the background is sketched here as well, within the context of the GONG project.

As far as solar structure is concerned the immediate purpose of the GONG project is to discover the variation of, say, density and $\Gamma_1$ throughout the Sun. This, however, is only a first step. Of more fundamental concern are

a) the application of this information to other types of stars
b) the knowledge about basic physics which may result from it.

Both of these objectives require that the solar data be interpreted by means of carefully computed solar models and their oscillation frequencies. The purpose of the GONG solar model project is to ensure that we know how to compute these models and frequencies.

To clarify this, it is useful to discuss briefly some properties of such computations. They require the definition of

1) the assumptions of the calculation
2) the physics to be used
3) the values of the parameters required.

These three items are not totally separate. In the case of traditional calculations 1) might include assumptions such as

1.1) spherical symmetry, i.e., no effect of rotation or magnetic field
1.2) perfect hydrostatic equilibrium
1.3) no mixing outside convection zones, as defined by the Schwarzschild criterion
1.4) no mass loss or accretion
1.5) no dynamical effect (i.e., turbulent pressure) of convection
1.6) evolution starts from a chemically homogeneous, static model
1.7) the oscillations are adiabatic.

The required physics under 2) includes
2.1) equation of state
2.2) opacity
2.3) energy generation, etc
2.4) description of convection energy transport
2.5) the surface boundary condition on the oscillations.

Finally the requirements under 3) are obvious. Given the complete definition of items 1) - 3), the computation of a solar model and its frequencies is a completely defined mathematical problem, with a unique solution. This is the basic forward problem, linking the unknowns, i.e., the physics and the assumptions, to the observables, i.e., the frequencies. The purpose of the model project is to make sure that the numerical procedures used to solve this mathematical problem are adequate. The precise definition of “adequacy” must depend on the observational precision achievable. Given the generally fairly low sensitivity of the model and frequencies to variations in the physics, and the very high expected accuracy of the observed frequencies, an ultimate goal of an accuracy of 0.1 \( \mu \text{Hz} \) over much of the \((\ell, \nu)\) plane does not seem excessive.

It is far from obvious that any of the present calculations of solar oscillations has an accuracy of even 1 \( \mu \text{Hz} \), in terms of the forward problem discussed above. This is the basic reason for carrying out an exercise like the present. It could be argued that for much of what we do this accuracy is not really needed. Thus the computations being made of the sensitivity of the models to changes in parameters are probably largely adequate, since most of the numerical errors may cancel. However for the analysis of the observed frequencies we must require the highest possible absolute accuracy of the computed frequencies. The Sun does not make numerical errors!

To make a comparative test of different programmes to compute solar models and oscillations it is essential that the same mathematical forward problem be treated, i.e., that the physics and the numerical constants are the same. The chief goal of these notes is to define this physics, and to give the proposed values of the constants. In addition some suggestions are made on the procedures for the model comparison, including the variables used and the data format.

1.1 Commonly used notation.

The notation employed is standard, but some of the more commonly used symbols are given here for convenience. Further notation is introduced later.

\( \begin{align*}
T: & \text{ temperature.} \\
p: & \text{ pressure.} \\
\rho: & \text{ density.} \\
u: & \text{ internal energy per unit volume.} \\
U: & \text{ internal energy per unit mass.} \\
n(H): & \text{ number of unionized hydrogen atoms per unit volume.} \\
N(H): & \text{ number of unionized hydrogen atoms per unit mass.} \\
n(\text{H}^+), n(\text{He}), n(\text{He}^+) & \text{ and } n(\text{He}^{++}) \text{ denote the number of ionized hydrogen atoms,} \\
& \text{and neutral, singly and doubly ionized helium atoms per unit volume, with a similar meaning of} \\
& N(\text{H}^+), N(\text{He}), N(\text{He}^+) \text{ and } N(\text{He}^{++}). \\
n_{\text{tot}}(\text{H}): & \text{ total number of hydrogen atoms per unit volume.} \\
N_{\text{tot}}(\text{H}): & \text{ total number of hydrogen atoms per unit mass.} \\
n_{\text{tot}}(\text{He}): & \text{ total number of helium atoms per unit volume.} \\
N_{\text{tot}}(\text{He}): & \text{ total number of helium atoms per unit mass.}
\end{align*} \)
\[ x(H^+) = N(H^+)/N_{\text{tot}}(H) \]: Degree of ionization of hydrogen.

Similarly \( x(He^{+}) \) and \( x(He^{++}) \) are the degrees of first and second ionization of helium.

\( \mu \): mean molecular weight.
\( \mu_n \): mean molecular weight of nuclei.
\( \mu_e \): mean molecular weight per electron.
\( n_e \): number of free electrons per unit volume.
\( N_e \): number of free electrons per unit mass.
\( X, Y, Z \): Abundances by mass of H, He and heavy elements.
\( \kappa \): opacity.
\( c \): rate of energy generation per unit mass.
\( r_X \): rate of change of \( X \).
\( c_p \): specific heat at constant pressure.
\( \delta \): \( \left( \frac{\partial \log \rho}{\partial \log T} \right)_p \).
\( \nabla_{\text{ad}} \): \( \left( \frac{\partial \log T}{\partial \log p} \right)_{\text{ad}} \).
\( \nabla \): \( \frac{\text{d} \log T}{\text{d} \log p} \).
\( m = m(r) \): mass interior to radius \( r \).
\( L = L(r) \): luminosity through sphere with radius \( r \).

\( k \): Boltzmann’s constant.
\( m_n \): atomic mass unit.
\( m_e \): electron mass.
\( e \): electron charge.
\( h \): Planck’s constant.
\( c \): speed of light.
\( a \): radiation density constant.
\( \sigma = ac/4 \): Stefan-Boltzmann constant.
\( G \): gravitational constant.
\( A_H \): atomic weight of hydrogen.
\( A_{He} \): atomic weight of helium.
\[ \left[ \frac{Z_i}{A_{i}} \right]_{h} \]: Average charge/mass ratio for heavy elements.
\[ \left[ \frac{1}{A_{i}} \right]_{h} \]: Average inverse mass for heavy elements.
\( < Z_i >_h \): Average nuclear charge for heavy elements.
\( \chi_H \): ionization potential for hydrogen.
\( \chi_{He} \): first ionization potential for helium.
\( \chi_{He^+} \): second ionization potential for helium.

1.2 The equations of stellar structure.

These are included here merely to ensure that the assumptions of the calculations are completely defined. We have

\[ \frac{dp}{dm} = -\frac{Gm}{4\pi r^4}, \]  
(1.1)
\[
\frac{dr}{dm} = \frac{1}{4\pi \rho r^2}, \quad (1.2)
\]

\[
\frac{dT}{dm} = \begin{cases} 
\frac{3kL}{64\pi c^2 r^4 T^3} & \text{in radiative case} \\
\nabla \frac{T}{p} \frac{dp}{dm} & \text{in convective case},
\end{cases} \quad (1.3)
\]

\[
\frac{dL}{dm} = \epsilon - \frac{dU}{dt} + \frac{p}{\rho^2} \frac{dp}{dt}, \quad (1.4)
\]

\[
\frac{dX}{dt} = rX. \quad (1.5)
\]

In equation (1.3) \( \nabla \) must be calculated from mixing-length theory in the convective case. This is discussed in Section 5. Note that the time derivatives in equations (1.4) and (1.5) are at fixed mass \( m \).

2. Equation of state.

As discussed in GONG Newsletter No 6, two levels of approximation to the equation of state will be used:

1) Assume all elements to be fully ionized.
2) Follow the partial ionization of H and He, but assume the heavy elements to be fully ionized.

In this case special precautions, beyond the simple Saha equation, are required to ensure full ionization in the solar interior. Here we use the “fudge” proposed by Eggleton et al. (1973; in the following EFF).

As a further simplification in both cases, we neglect radiation pressure and partial degeneracy. Neither effect is substantial in the solar case, but they are significant at the level of accuracy desired in the comparison. Hence a common policy is required, complete neglect being the simplest. When partial ionization is dealt with, it is also assumed that all atoms are in the ground state, so that the partition function is approximated by the ground state statistical weight.

2.1 The level 1 equation of state.

Pressure:

\[
p = \frac{kT}{\mu m_u}, \quad (2.1)
\]

where

\[
\mu^{-1} = 2 \frac{X}{A_H} + 3 \frac{Y}{A_{He}} + \left[ \frac{Z_i}{A_i} + \frac{1}{2} \right] \frac{Z}{Z}. \quad (2.2)
\]

Internal energy:

\[
u = \frac{3}{2} p. \quad (2.3)
\]

This defines the equation of state precisely.

2.2 The level 2 equation of state.

\[
p = \frac{kT}{\mu m_u} + \Delta p. \quad (2.4)
\]
Internal energy:

\[ u = \frac{3}{2} \frac{kpT}{\mu m_u} + u_1 + \Delta u . \]  

Here \( \Delta p \) and \( \Delta u \) are corrections to pressure and internal energy, caused by the EFF treatment of pressure ionization and presented below; \( u_1 \) is the ionization energy.

The mean molecular weight is given by

\[ \mu^{-1} = \rho^{-1} + \mu_e^{-1} , \]

where

\[ \rho^{-1} = \frac{X}{A_H} + \frac{Y}{A_{He}} + \left[ \frac{1}{A_i} \right]_h Z , \]

and

\[ \mu_e^{-1} = \frac{n_e m_u}{\rho} = \frac{x(H^+)}{A_H} + \frac{[x(He^+) + 2x(He^{++})]}{A_{He}} + \left[ \frac{Z_i}{A_i} \right]_h Z . \]

In the ionization energy, we take the zero point in the unionized state, and omit the (constant) contribution from the heavy elements. Hence

\[ u_1 = \frac{\rho}{m_u} \left\{ \chi_H x(H^+) \frac{X}{A_H} + [\chi_{He} x(He^+) + (\chi_{He} + \chi_{He+}) x(He^{++})] \frac{Y}{A_{He}} \right\} . \]

2.2.1 Ionization calculation.

Using explicitly the assumption that the partition functions are given by the ground state statistical weights, the Saha equations are

\[ \frac{n(H^+)}{n(H)} = \left( \frac{2 \pi k T m_e}{\hbar^2} \right)^{3/2} \frac{1}{n_e} \exp(-\chi_H/kT + \Delta \mu) , \]

\[ \frac{n(He^+)}{n(He)} = 4 \left( \frac{2 \pi k T m_e}{\hbar^2} \right)^{3/2} \frac{1}{n_e} \exp(-\chi_{He}/kT + \Delta \mu) , \]

\[ \frac{n(He^{++})}{n(He^+)} = \left( \frac{2 \pi k T m_e}{\hbar^2} \right)^{3/2} \frac{1}{n_e} \exp(-\chi_{He^+}/kT + \Delta \mu) . \]

Here \( \Delta \mu \) is the EFF correction that ensures full ionization in the interior.

From these quantities one obtains the degrees of ionization \( x(H) \), \( x(He^+) \) and \( x(He^{++}) \).

2.2.2 EFF prescription.

The correction \( \Delta \mu \) is written as

\[ \Delta \mu = C_F a_0^3 (kT + 20 \chi_H Z^2) \frac{n_e}{kT} . \]
Here

\[ a_0 = \frac{\hbar^2}{4\pi^2 m_e e^2} \]  \hspace{1cm} (2.14)

is the Bohr radius (assuming \( e \) given in ESU), and

\[ \bar{Z} = X + 2Y + \left< Z_i \right>_h Z \]  \hspace{1cm} (2.15)

is the mean charge per nucleus. Also the constant \( C_F \) is a correction factor, relative to the original EFF treatment, to allow adjusting the approximation to give the best possible fit to more precise treatments of the equation of state.

With this \( \Delta \mu \), thermodynamic consistency is ensured by including the correction to \( p \)

\[ \Delta p = \frac{1}{2} C_F a_0^3 \left( kT \right) \chi_H (n_{e0}^2 - n_e^2), \]  \hspace{1cm} (2.16)

and the correction to \( u \)

\[ \Delta u = 10 C_F a_0^3 \chi_H (n_{e0}^2 - n_e^2), \]  \hspace{1cm} (2.17)

where

\[ n_{e0} = \left( \frac{X}{A_H} + 2 \frac{Y}{A_{He}} + \left[ \frac{Z_i}{A_i} \right]_h Z \right) \frac{\rho}{m_u} \]  \hspace{1cm} (2.18)

is the total number of electrons, bound or free.

2.2.3 Computational procedures.

At fixed \( T \) and \( \rho \) equations (2.8) and (2.10) - (2.13) provide an implicit equation that may be solved for \( n_e \). This requires iteration. This is not qualitatively different from the usual situation when the simple Saha equation is used; the inclusion of the term in \( \Delta \mu \) introduces a slight complication, but apparently does not change the convergence properties of the iteration significantly. Given \( n_e \), the remaining required thermodynamic quantities can then be obtained.

Alternatively, as suggested by EFF, the thermodynamic state can be regarded as a function of the variables \( T \) and \( n_e \). Given \( T \) and \( n_e \), \( \Delta \mu \) and the degrees of ionization can be calculated explicitly. Equation (2.8) can then be used to determine \( \rho \). Thus the equation of state is an explicit function of \( T \) and \( n_e \), with the choice made for \( \Delta \mu \). This is the motivation for that choice.

To facilitate the treatment of partial degeneracy, EFF work in terms of the variables \( \psi \) or \( f \) instead of \( n_e \), where \( \psi \) is the electron degeneracy parameter. In the present case, where partial degeneracy is neglected, a modified definition of \( f \) is more appropriate. This corresponds to the limit of no degeneracy in the EFF case. Thus I propose to introduce \( f \) by

\[ \psi = 2 + \log(f/4). \]  \hspace{1cm} (2.19)

Then, in the same form as EFF, the density can be expressed by

\[ \frac{\lambda_e^3 n_e}{8\pi} = \frac{\lambda_e^3 \rho}{8\pi \mu_e m_u} \equiv \rho^*_e = \rho_0 f T^* f^{3/2}, \]  \hspace{1cm} (2.20)

where \( \lambda_e = \hbar/m_e c \) is the electron Compton wavelength, and \( T^*_e = kT/m_e c^2 \). Here the constant \( \rho_0^* \) is given by

\[ \rho_0^* = \frac{\sqrt{2\pi \mu_e^3}}{8} = 2.31520211755... \]  \hspace{1cm} (2.21)
where, in this equation only, \( e \) denotes the base of the natural logarithm.

In terms of \( \psi \), the Saha equations can be written

\[
\frac{n(H^+)}{n(H)} = \frac{1}{2} \exp(-\psi - \chi_{H}/kT + \Delta\mu),
\]

(2.22)

\[
\frac{n(He^+)}{n(He)} = 2 \exp(-\psi - \chi_{He}/kT + \Delta\mu),
\]

(2.23)

\[
\frac{n(He^{++})}{n(He^+)} = \frac{1}{2} \exp(-\psi - \chi_{He^+}/kT + \Delta\mu).
\]

(2.24)

EFF present explicit formulae for the equation of state in terms of \( f \) and \( T \), including an approximate treatment of partial degeneracy. I use this formulation in my normal computation of stellar models. To simplify the comparison, my simplified calculation uses the \( f \) defined by the expressions above.

2.3 Values of specific constants.

Apart from the natural constants (given in Section 7 below), the expressions for the equation of state contain some specific constants, whose values must be chosen. In the spirit of the project, it is essential that common values be used; these should be reasonable, but it is not essential that optimum values be chosen. I propose the following:

\[
\begin{align*}
\left[ \begin{array}{c}
Z_i \\
A_i \\
1 \\
\hat{Z_i}
\end{array} \right]_{h} &= 0.5 \\
A_i &= 0.0625 \\
\langle Z_i \rangle_{h} &= 8 \\
C_F &= 15
\end{align*}
\]

The averages for the heavy elements are based on oxygen, which is the most abundant element. In a more realistic computation they should evidently be replaced by actual averages over the assumed relative abundances of the heavy elements. The value of the correction factor \( C_F \) was obtained by Christensen-Dalsgaard (1978) by fitting to a more detailed equation of state.

3. Opacity.

The opacity is approximated by a two-term power law expression, on the form

\[
\kappa^{-1} = \kappa_e^{-1} + \kappa_i^{-1}.
\]

(3.1)

where

\[
\kappa_e = c_e \rho^{m_e} T^{n_e},
\]

(3.2)

and

\[
\kappa_i = c_i \rho^{m_i} T^{n_i}.
\]

(3.3)

This form was first suggested by M. Gabriel. With suitable choice of coefficients, \( \kappa \approx \kappa_e \) near the surface, and \( \kappa \approx \kappa_i \) beneath the convection zone, the transition taking place within the convection zone, where the value of the opacity is immaterial.
The appropriate values of the coefficients were determined by F. Pérez Hernández by least squares fitting to the opacity in a model of the present Sun. The results, to be used for the model calculation, are

\begin{align*}
    c_e &= 1.6236784 \times 10^{-33}.
    m_e &= 0.407895.
    n_e &= 9.28289.
    a_1 &= 7.1548412 \times 10^{13}.
    m_i &= 0.138316.
    n_i &= -1.97541.
\end{align*}

Here, as elsewhere, cgs units are used.

4. Energy production and rate of change of X.

We approximate \( \epsilon \) and \( r_X \) on the form obtained from the pp reaction rate, adjusting the coefficients to fit the complete expressions. Only the changes in hydrogen (and helium) abundances are followed. All other elements are assumed to be in nuclear equilibrium, and the changes in their abundances are not taken into account in the equation of state.

I write \( \epsilon \) as

\[ \epsilon = Q_e a_{11} \frac{X^2}{2m_a} \rho T_9^{-2/3} \exp(-bT_9^{-1/3}) , \tag{4.1} \]

and

\[ r_X = \frac{dX}{dt} = -R_e a_{11} \frac{X^2}{2} \rho T_9^{-2/3} \exp(-bT_9^{-1/3}) . \tag{4.2} \]

Here \( T_9 = T/(10^9 K) \). F. Pérez Hernández made a least-squares fit of a similar expression to an accurate calculation of \( \epsilon \) and \( r_X \). This fit, however, had the unpleasant property of causing convective cores in early models in a sequence leading to a model of the present Sun. Thus I have modified the coefficients somewhat. The proposed values, in cgs units, are

\begin{align*}
    a_{11} &= 4.21 \times 10^{-15} \\
    Q_e &= 6.5 \times 10^{-5} \\
    b &= 3.6 \\
    R_e &= 6.5.
\end{align*}

5. The temperature gradient.

Radiative transfer is treated in the diffusion approximation. Hence, in regions that are stable towards convection, the temperature gradient is given by

\[ \frac{dT}{dr} = -K^{-1} \frac{L}{4\pi r^2} , \tag{5.1} \]

where

\[ K = \frac{4acT^3}{3\kappa \rho} \tag{5.2} \]

is the conductivity.
In convectively unstable regions, energy transport is treated using mixing-length theory. We again assume that radiation is treated with the diffusion approximation, and furthermore that the convective elements are optically thick. For completeness I also note that effects of turbulent pressure are not taken into account.

Among the several different equivalent descriptions of mixing length theory we follow the one of Henyey, Vardya & Bodenheimer (1965). Thus we write

\[ \nabla - \nabla_{ad} = \frac{\gamma(\gamma + 1)}{B}, \tag{5.3} \]

where \( \gamma \) satisfies the equation

\[ \phi \gamma^3 + \gamma(\gamma + 1) = B(\nabla_R - \nabla_{ad}). \tag{5.4} \]

Here

\[ B = \frac{\xi \ell^4 g \delta (pc_p)^2}{H_p R^2}, \tag{5.5} \]

where the mixing length \( \ell \) is

\[ \ell = \alpha H_p, \tag{5.6} \]

the pressure scale height \( H_p \) being

\[ H_p = \frac{p}{\gamma \rho}, \tag{5.7} \]

where

\[ g = \frac{G m}{r^2}. \tag{5.8} \]

Also the radiative temperature gradient \( \nabla_R \) is given by

\[ \nabla_R = \frac{L H_p}{4 \pi r^2 K T}. \tag{5.9} \]

The quantities \( \phi \) in equation (5.4) and \( \xi \) in equation (5.5) are geometrical constants that are essentially arbitrary. In the notation of Henyey et al.,

\[ \phi = \frac{3}{4y}, \quad \xi = \frac{4y^2}{9\nu}, \tag{5.10} \]

where \( y \) and \( \nu \) are other constants, for which Henyey et al. recommend the values 0.076 and 8, respectively. Alternatively, in the notation of Gough (1977),

\[ \phi = (2\Phi^{1/2} \eta)^{-1}, \quad \xi = \frac{\eta^2}{4}, \tag{5.11} \]

where \( \Phi \) and \( \eta \) are yet another set of constants; Gough notes that for \( \Phi = 2, \eta = \sqrt{2}/9 \), the expressions given here reduces to those of Böhm-Vitense (1958).

To guide the choice of parameters in the calculation, it is instructive to consider the asymptotic dependence of \( \nabla - \nabla_{ad} \) on \( \phi, \xi \) and \( \alpha \). The relevant limit in the solar case is \( B(\nabla_R - \nabla_{ad}) \gg 1 \). Here equation (5.4) has the approximate solution

\[ \gamma \approx \left( \frac{B}{\phi} \right)^{1/3} (\nabla_R - \nabla_{ad})^{1/3}, \tag{5.12} \]
and hence equation (5.3) gives

\[ \nabla - \nabla_{ad} \approx (\phi \xi^{1/2} \alpha^2)^{-2/3} B_0^{-1/3} (\nabla_R - \nabla_{ad})^{2/3}, \]

(5.13)

where

\[ B_0 = \frac{H_0^3 g \delta (\rho c_p)^2}{K^2}. \]

(5.14)

Thus here the arbitrary constants \( \phi, \xi \) and \( \alpha \) only enter in the combination \( \phi \xi^{1/2} \alpha^2 \). In calculations of models of the present Sun \( \alpha \) is always adjusted to obtain the correct radius. Equation (5.13) shows that any change in \( \phi \) or \( \xi \) would require a corresponding change in \( \alpha \), leaving \( \phi \xi^{1/2} \alpha^2 \) and hence the resulting model essentially unchanged. Thus, at this level of accuracy, the model is independent of the choice of \( \phi \) and \( \xi \). This is consistent with the discussion of Chan's presentation at the Yale workshop, in GONG Newsletter No. 6, p. 7. It might also be noticed that in terms of the quantities used by Henyey et al. rm and Gough,

\[ \phi \xi^{1/2} = (2\sqrt{\nu})^{-1} = (4\Phi^{1/2})^{-1}. \]

(5.15)

Thus the proposed value of \( \nu = 8 \), and the value of \( \Phi = 2 \) leading to the Böhm-Vitense form, both correspond to \( \phi \xi^{1/2} = \sqrt{2}/8 \).

For the present model comparison, a definite choice has to be made of \( \phi \) and \( \xi \). I propose

\[ \phi = \frac{9}{4}, \]

\[ \xi = \frac{1}{162}, \]

corresponding to the Böhm-Vitense form.

6. Boundary conditions.

6.1 Surface conditions.

To avoid the complication of a solar atmosphere, we use the following very simple surface boundary conditions:

\[ p = \frac{\beta g}{\kappa} \quad \text{at} \quad r = R, \]

(6.1)

\[ L = \lambda 4\pi R^2 \sigma T^4 \quad \text{at} \quad r = R. \]

(6.2)

Here \( \beta \) and \( \lambda \) are constants that must be chosen appropriately.

For \( \lambda = 1 \) the condition (6.2) clearly requires that the solar surface be at the point where \( T = T_{\text{eff}} \), the effective temperature. This is the most natural condition. However, when the level 1 EOS is used (assuming full ionization everywhere) the modifications to the outer parts of the model are so severe that changes to the boundary conditions are required. These can be obtained by changing \( \beta \) and \( \lambda \). A value of \( \lambda = 6 \) is suitable. In this case also the convection zone extends to the surface and is nearly adiabatic everywhere. This is primarily caused by the fact that the density is substantially higher near the surface than in the normal models, the opacity correspondingly higher and convective energy transport more efficient. The normal determination of \( \alpha \) by fixing the solar radius depends on the variation with \( \alpha \) in the entropy jump between the solar atmosphere and the nearly adiabatic part of the convection zone. In the present case, where convection is almost adiabatic everywhere, the entropy jump is very small and the model is thus nearly insensitive to the value of \( \alpha \). In this case, therefore, I propose to fix \( \alpha \), and adjust \( \beta \) so as to obtain the correct radius.
In summary, I make the following recommendations:

For the level 1 EOS.
\( \lambda = 6. \)
\( \beta \) adjusted to obtain correct radius.

For the level 2 EOS.
\( \lambda = 1. \)
\( \beta = 1. \)

6.2 Central conditions.

The centre is a regular singular point of the structure equations. Here conditions must be specified which ensure regular behaviour of the solution. These conditions depend on the precise formulation used for solving the equations. Thus the specification of the conditions must be regarded as part of the numerical procedure, which is what is being tested in this project.

7. Fundamental constants.

With the exception of \( G \), discussed below, the values of the fundamental constants are taken from the 1986 CODATA list (cf. Cohen & Taylor 1987). The atomic weights are based on mass excesses in Lederer et al. (1978), and the ionization potentials come from Moore (1971). The units are cgs, except where otherwise noted.

\[
\begin{align*}
\text{Boltzmann's constant} & \quad k & 1.380658 \times 10^{-16} \\
\text{atomic mass unit} & \quad m_u & 1.6605402 \times 10^{-24} \\
\text{electron mass} & \quad m_e & 9.1093897 \times 10^{-28} \\
\text{electron charge} & \quad e & 1.60217733 \times 10^{-19} \text{ C} \\
\text{Planck's constant} & \quad h & 6.6260755 \times 10^{-27} \\
\text{speed of light} & \quad c & 2.99792458 \times 10^{10} \\
\text{radiation density constant} & \quad a & 7.565914 \times 10^{-15} \\
\text{Stefan-Boltzmann constant} & \quad \sigma = ac/4 & 5.67051 \times 10^{-5} \\
\text{Conversion from eV to erg} & \quad \text{1 eV} = & 1.60217733 \times 10^{-12} \text{ erg} \\
\text{gravitational constant} & \quad G & 6.672320 \times 10^{-8} \\
\text{Solar mass} & \quad M_\odot & 1.989 \times 10^{33} \\
\text{atomic weight of hydrogen} & \quad A_H & 1.007825 \\
\text{atomic weight of helium} & \quad A_{\text{He}} & 4.002603 \\
\text{ionization potential for hydrogen} & \quad \chi_H & 13.595 \text{ eV} \\
\text{first ionization potential for helium} & \quad \chi_{\text{He}} & 24.580 \text{ eV} \\
\text{second ionization potential for helium} & \quad \chi_{\text{He}^+} & 54.403 \text{ eV}
\end{align*}
\]
Notes:

i) The CODATA value of the gravitational constant is \( G = (6.67259 \pm 0.00085) \times 10^{-8} \). \( G \) and \( M_\odot \) are connected by \( G M_\odot = 1.32712438 \times 10^{26} \) (in cgs units), from solar system dynamics. Thus the CODATA value of \( G \) corresponds to \( M_\odot = 1.988919 \times 10^{33} \). However the value \( M_\odot = 1.989 \times 10^{33} \), which is commonly used, corresponds to \( G = 6.672320 \times 10^{-8} \), which is well within the error bars of the CODATA value. Thus I propose to use the values given in the table.

ii) Of the values given above, \( a \) and \( \sigma \) are not fundamental, in the sense that they are given directly in terms of the remaining constants as

\[
a = \frac{8\pi^5 k^4}{15c^3 h^3}, \tag{7.1}
\]

\[
\sigma = \frac{ac}{4}. \tag{7.2}
\]

Their values are included here for convenience. However computationally it might be preferable to recompute them in the programme, to allow easier future modifications of the constants.

8. Notes on the computation.

To simplify the calculation, we ignore the pre-main sequence contraction phase, and start the evolution at a chemically homogeneous, static model, characterized by the initial hydrogen abundance \( X_0 \).* The object of the computation is to evolve models of this type to the age \( t_\odot \) of the present Sun, adjusting the value of \( X_0 \) and the mixing-length parameter \( \alpha \) (or some other parameter) so as to obtain the solar radius \( R_\odot \) and luminosity \( L_\odot \) at the age \( t_\odot \). With the definitions given in the preceding sections, this is a precise numerical problem. The purpose of the project is then obviously to test the methods used to solve this problem. These methods depend on the choice of the number and distribution of mesh points, the choice of time step, and the treatment of the central singularity. At a more trivial, but no less important, level the results depend on proper programming and proper definition of the constants entering into the computation.

8.1 The iteration for \( R \) and \( L_s \).

In normal solar evolution calculations, \( X_0 \) and \( \alpha \) are chosen so that the model satisfies the constraints

\[
R(t_\odot) = R_\odot, \tag{8.1}
\]

\[
L_s(t_\odot) = L_\odot, \tag{8.2}
\]

where \( R \) and \( L_s \) are the surface radius and luminosity. This requires iteration. I have found that the following set of derivatives (Christensen-Dalsgaard 1982) works reasonably well, in ensuring adequate convergence:

\[
\left( \frac{\partial \log \alpha}{\partial \log L_s} \right)_R = 1.314 \quad \left( \frac{\partial \log \alpha}{\partial \log R} \right)_{L_s} = -4.918
\]

\[
\left( \frac{\partial \log X_0}{\partial \log L_s} \right)_R = -0.150 \quad \left( \frac{\partial \log X_0}{\partial \log R} \right)_{L_s} = -0.040. \tag{8.3}
\]

* i.e., computed without the time derivative terms in equation (1.4) for \( L \).
All quantities are evaluated at the present age of the Sun, and the derivatives assume that a full evolution calculation is carried out. Thus, if \( \delta \log L_s = \log L_\odot - \log L_s \) and \( \delta \log R = \log R_\odot - \log R \) are the errors in \( L_s \) and \( R \), the corrections to \( \alpha \) and \( X_0 \) are obtained from

\[
\delta \log \alpha = \left( \frac{\partial \log \alpha}{\partial \log L_s} \right)_R \delta \log L_s + \left( \frac{\partial \log \alpha}{\partial \log R} \right)_{L_s} \delta \log R ,
\]

\[
\delta \log X_0 = \left( \frac{\partial \log X_0}{\partial \log L_s} \right)_R \delta \log L_s + \left( \frac{\partial \log X_0}{\partial \log R} \right)_{L_s} \delta \log R .
\]

In fact the iteration process is built into my evolution code and is performed automatically.

As discussed in Section 6, when full ionization everywhere is assumed, convection is nearly adiabatic everywhere, and thus the structure of the model is nearly insensitive to \( \alpha \). In this case, therefore, it is not possible to obtain the correct radius by adjusting \( \alpha \). Instead, we fix \( \alpha \) and adjust the constant \( \beta \) entering in the surface pressure condition (cf. equation (6.1)). In this case the corresponding derivatives are

\[
\left( \frac{\partial \log \beta}{\partial \log L_s} \right)_R = 2.723 \quad \left( \frac{\partial \log \beta}{\partial \log R} \right)_{L_s} = -10.97
\]

\[
\left( \frac{\partial \log X_0}{\partial \log L_s} \right)_R = -0.1516 \quad \left( \frac{\partial \log X_0}{\partial \log R} \right)_{L_s} = -0.046 .
\]

Note that equations (8.1) and (8.2) should be satisfied to substantially better than the desired accuracy of the comparison. I suggest that the goal should be a relative error in these equations of no more than \( 10^{-5} \).

8.2 Suggested computations.

It is evidently essential to define a fixed set of models to compute and compare. I suggest to exchange both initial, chemically homogeneous (i.e., ZAMS) models, to test just the basic structure calculation, and evolved, calibrated models of the present Sun. Specifically I propose that we consider the following cases, all with \( Z = 0.02 \):

**Level 1 EOS:**

1.a) ZAMS model with \( X_0 = 0.733, \alpha = 1.6, \beta = 7.22, \) and \( \lambda = 6 \). (This is, according to my results, fairly close to a model evolving into a properly calibrated model of the present Sun).

1.b) Calibrated model of the present Sun, with \( \alpha = 1.6 \) and \( \lambda = 6 \). \( X_0 \) and \( \beta \) are determined through calibration.

**Level 2 EOS.**

2.a) ZAMS model with \( X_0 = 0.733, \alpha = 1.74, \beta = 1, \) and \( \lambda = 1 \). (This is, according to my results, fairly close to a model evolving into a properly calibrated model of the present Sun).

2.b) Calibrated model of the present Sun, with \( \beta = 1 \) and \( \lambda = 1 \). \( X_0 \) and \( \alpha \) are determined through calibration.
Level 3 physics.

3.b) Calibrated model of the present Sun. Note that, in accordance with GONG Newsletter No 6, Level 3 physics refers to the currently used physics in the evolution programmes. This case, therefore, deviates from the constraint of common physics. It is included here to start the comparison of the actual “best” models. To facilitate the estimates of the effects of different physics it would be highly desirable if this model were computed with similar numerical procedures to the models under Levels 1 and 2.

The procedures for data exchange are discussed in more detail in Section 9.

8.3 Properties of the present Sun.

We must specify the values to use for the present Sun. These clearly also have to be the same for all the calculations. I propose the following:

\[ t_\odot = 4.75 \times 10^9 \text{ years.} \]
\[ R_\odot = 6.9599 \times 10^{10} \text{ cm.} \]
\[ L_\odot = 3.846 \times 10^{33} \text{ erg s}^{-1}. \]

The value of \( t_\odot \) is based on taking the age of the Earth as \( 4.55 \times 10^9 \) years (Patterson, 1956), and adding a suitable small amount. \( R_\odot \) comes from Allen (1973), and is consistent with Parkinson, Morrison & Stephenson (1980). Finally \( L_\odot \) is based on an eye-ball average of the irradiance results obtained by Willson et al. (1986). It is obvious that all these values can, and eventually should, be questioned. For the present purpose, however, it is essential that the same values be used by everybody, and those proposed above appears reasonable.

In the Level 3 case \( R_\odot \) should, to be definite, refer to the radius at the point where \( T = T_{\text{eff}}. \)


The exchange of models will take place by means of ASCII files, to avoid problems with different binary formats. This makes it possible to send the data electronically without problems (other than caused by the amount), and should minimize the difficulty in using magnetic tapes.

For each model the file consists of a header, with descriptive information, a set of global variables, and a set of variables given at each mesh point.

The set of variables is likely to develop, particularly in the context of the use of the format for transferring realistic solar and stellar models. The actual set used is defined by the variable \texttt{vers} included in the header to the file. In addition, the total number of global parameters, meshpoints and variables at each meshpoint are given in the header, so that the information required to read the file is available.

9.1 Text header.

The first record should contain the name of the model, its date, and an identification of its origin (such as name or institute). In addition the header may contain text further describing the calculation, and information about the remaining data.

An (purely fictitious) example of a header may be the following:

\texttt{L1.013.PRES.880226.AARHUS}  
\texttt{Level 1 model of present Sun. Correcting error}
in opacity routine reported Feb. 24.

9.2 Global parameters.

1: \( M \) (total mass).
2: \( R \) (photospheric radius).
3: \( L_s \) (surface luminosity).
4: \( Z \) (heavy element abundance).
5: \( X_0 \) (initial hydrogen abundance).
6: \( \alpha = \ell/H_p \) (mixing-length parameter)
7: \( \phi \) (another convection theory parameter)
8: \( \xi \) (yet another convection theory parameter)
9: \( \beta \) (parameter in surface pressure condition)
10: \( \lambda \) (parameter in surface luminosity condition)
11: \( \frac{R^2 d^2 p_c}{d r^2} \) (at centre).
12: \( \frac{R^2 d^2 \rho_c}{\rho_c \ d r^2} \) (at centre).
13: Model age (in years) (added 30/7/96).
14 - 15: Unused, so far.

The parameters \( \beta \) and \( \lambda \) are defined in equations (6.1) and (6.2), whereas \( \phi \) and \( \xi \) are defined in equations (5.4) and (5.5). The second derivatives in variables 11 and 12 may be needed in the central boundary conditions for the oscillation calculations.

9.3 Model variables at each mesh point.

The current set of variables, as defined in July 1996, is characterized by having \texttt{ivers} = 250. The notation is the same as introduced in section 1.1, unless otherwise noted.

1: \( r \)
2: \( \ln q, \quad q = m/M \)
3: \( T \)
4: \( p \)
5: \( \rho \)
6: \( X \)
7: \( L(r) \)
8: \( \kappa \)
9: \( \epsilon \)
10: \( \Gamma_1 \)
11: \( \nabla_{ad} \)
12: \( \delta = - \left( \frac{\partial \log \rho}{\partial \log T} \right)_p \)
13: \( c_p \)
14: \( \mu_{c^{-1}} \)
15: \( \frac{1}{\Gamma_1} \frac{d \log p}{d \log \rho} = \frac{d \log \rho}{d \log r} \)
16: \( r_X \)
17: \( Z \)
18: \( R - r \)
19: \(\epsilon_s\) (the gravitational energy release)
20: \(L_s\) (local gravitational luminosity; this has only been included in J. Reiter's models so far)
21: \(X(\text{He}^3)\) \(^{3}\text{He}\) abundance by mass
22: \(X(\text{C}^{12})\) \(^{12}\text{C}\) abundance by mass
23: \(X(\text{C}^{13})\) \(^{13}\text{C}\) abundance by mass
24: \(X(\text{N}^{14})\) \(^{14}\text{N}\) abundance by mass
25: \(X(\text{O}^{16})\) \(^{16}\text{O}\) abundance by mass
26: \(\left(\frac{\partial \ln \Gamma_1}{\partial \ln \rho}\right)_{p,Y}\)
27: \(\left(\frac{\partial \ln \Gamma_1}{\partial \ln \rho}\right)_{\rho,Y}\)
28: \(\left(\frac{\partial \ln \Gamma_1}{\partial Y}\right)_{p,\rho}\)
29 – 30: Currently not used.

Notes:

i) In variable 2, \(\ln q\) is used instead of \(m\) to give a better indication of variation close to the surface. Here \("\ln\"\) is natural logarithm.

ii) \(\mu_e^{-1}\) is included to give some indication of the ionization state. For the Level 1 equation of state it may be omitted.

iii) Variable 18 was introduced Aug. 6 1993, to avoid problems with interpolation in \(r\) near surface.
(For versions before iwers = 210, but after that date, \(R - r\) was set in variable 17.)

iv) The derivatives of \(\Gamma_1\) were introduced principally to allow calculation of kernels involving \(Y\), in realistic models; they are less essential for the comparison of models.

v) Calculations that do not follow all details of the CNO cycle may not have available some of the CNO abundances. In that case, the corresponding abundances can be set to zero. (An example might be a calculation following the conversion of \(^{16}\text{O}\) into \(^{14}\text{N}\) but ignoring the details of carbon burning into \(^{14}\text{N}\).)

vi) More generally, for a given code there might be variables that are difficult to obtain. In that case, the above numbering should be maintained, but the missing variables may be set identically to zero.

The present set of variables has been chosen to give a reasonably comprehensive basis for comparing evolution models, and to be adequate for the computation of adiabatic oscillations. In particular these variables should define completely the stellar structure equations, and so permit a check of the accuracy to which the equations are satisfied. For a more detailed comparison even more variables may be needed (such as ionization levels), but I suggest that that be arranged separately.

For non-adiabatic calculations, which we should eventually get to, more variables are certainly needed. Typical examples are the derivatives of \(\kappa\) and \(\epsilon\) with respect to \(p\) and \(T\), and possibly variables relating to the perturbation of the convective flux. They can be included later, by extending the basic set given above.

9.4 Format for data transfer.

The data exchange should be carried out by means of formatted ASCII files. I suggest the following structure:

**Record 1**: Name of model (as a character string)

**Record 2 – 4**: Explanatory text, in free format
Record 5: nn, icost , ivar, ivers  
Record 6 - 8: glob(i), i = 1, ..., icost  
Record 9 - : var(i,n), i = 1, ..., ivar, n = 1, ... nn.

Here nn is the number of mesh points in the model, icost is the number of global variables (given in the array glob), and ivar is the number of variables at each mesh point (given in the array var). Thus with the present set icost = 15 and ivar = 30, but there is room for expansion. The version number ivers was discussed above.

I suggest writing the integer variable Record 5 with the format

4i10

and the real variable Records 6 and up with the format

1p5e16.9

I have been principally responsible for the model comparison so far and am quite prepared to continue to do so. Since relatively modest amounts of data are involved, sending models by means of electronic mail, or using anonymous ftp, is quite feasible; alternatives, such as PC diskettes, might also be considered.

References:


Appendix: Summary of earlier versions

- **Version 100**: var(1) - var(16) defined as above. In some models var(17) was set to $R - r$, and var(19) and var(20) were set as above; otherwise these variables were set to zero; ivar = 20.

- **Version 200**: var(1) - var(16) and var(19) - var(25) defined as above, var(17) set to $R - r$; ivar = 25.

- **Version 210**: var(1) - var(25) defined as above; ivar = 25.